Quality-aware and Fine-grained Incentive Mechanisms for Mobile Crowdsensing

Jing Wang, Jian Tang, Dejun Yang, Erica Wang and Guoliang Xue

***Abstract*—Limited research efforts have been made for Mobile CrowdSensing (MCS) to address quality of the recruited crowd, i.e., quality of services/data each individual mobile user and the whole crowd are potentially capable of providing, which is the main focus of the paper. Moreover, to improve flexibility and effectiveness, we consider fine-grained MCS, in which each sensing task is divided into multiple subtasks and a mobile user may make contributions to multiple subtasks. In this paper, we first introduce mathematical models for characterizing the quality of a recruited crowd for different sensing applications. Based on these models, we present a novel auction formulation for quality-aware and fine-grained MCS, which minimizes the expected expenditure subject to the quality requirement of each subtask. Then we discuss how to achieve the optimal expected expenditure, and present a practical incentive mechanism to solve the auction problem, which is shown to have the desirable properties of truthfulness, individual rationality and computational efficiency. We conducted trace-driven simulation using the mobility dataset of San Francisco taxies. Extensive simulation results show the proposed incentive mechanism achieves noticeable expenditure savings compared to two well-designed baseline methods, and moreover, it produces close-to-optimal solutions.**

***Index Terms*—Mobile Crowdsensing, Smartphones, Incentive Mechanism, Auction, Quality of Crowd**

I. Introduction

Beyond communications, mobile phones have been playing a key role in many aspects of people's daily life, including computing, entertainment, etc. However, some mobile users may not fully realize that most smartphones (such as iPhone 6, Nexus 6, Lumia series, etc) are equipped with a rich set of powerful embedded sensors, such as camera, GPS, WiFi/3G/4G interface, accelerometer, digital compass, gyroscope, microphone, etc. Additionally, the emergence of wearable devices (such as smart watch, Fitbit, Sensordrone [20], etc) significantly extends the sensing capabilities of smartphones. Most wearable devices can be connected to a smartphone via a network interface (such as Bluetooth) for data exchange. Ubiquitous mobile sensors have led to many attractive sensing applications in various domains [13], including environmental monitoring, social networking, healthcare, transportation, safety, etc.

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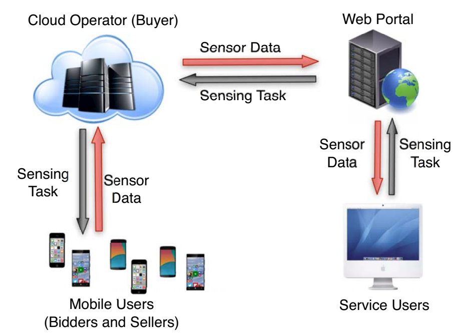

Fig. 1. An MCS System

Recently, *Mobile CrowdSensing* (MCS) have been gaining increasing popularity. As shown in Fig. 1, we consider a general-purpose MCS system [21], such as PRISM [1] and Medusa [19]. A service user can make a sensing service request via a web portal. The request is then analyzed by the cloud operator, which will use an incentive mechanism to recruit a sensing crowd (a set of mobile users) and distribute the request to them. Then their smartphones will perform the corresponding sensing activities and report sensor data to the cloud operator. The cloud operator will aggregate and analyze sensor data, and then send results back to the service user through the web portal.

While participating in MCS, there is usually a *cost* occurring to a mobile user. For example, performing sensing activities consumes energy from a smartphone. So the mobile user may want to earn certain credits (e.g., money) to compensate for his/her energy loss. Most sensing tasks are location-dependent, which may require mobile users to travel to or around certain areas, leading to certain costs such as transportation. Furthermore, mobile users usually won't be willing to share their privacy while undertaking sensing tasks if there are no satisfactory rewards. Based on the above observations, we are motivated to consider a *reverse auction* based incentive mechanism to enable fair pricing between the cloud operator and mobile users in MCS. As illustrated in Fig. 1, after receiving a sensing task from a service user, the cloud operator (the buyer of sensor data) announces it to mobile users. Mobile users (bidders, sellers and service providers) offer their bids for undertaking the task and selling their sensor data. Based on the bids, the cloud operator will selectively determine winners and after collecting sensor data from winners, it will make payments to them. Auction mechanism design is crucial for

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supporting MCS, because the trading rules between the buyer (the cloud operator) and the sellers (mobile users) heavily depend on it. Specifically, among all the behavior characteristics of bidders, *truthfulness* [17] and *individual rationality* [10] are of special interest and most desirable in MCS. An auction mechanism is truthful if a bidder will not increase its payoff by submitting any other bids instead of his/her true values. An auction without truthfulness will be vulnerable to market manipulation and produce very poor outcomes [8]. An auction mechanism is individually rational if the payoff of every bidder is not negative by bidding his/her true values.

Besides the cost, the success of a crowdsourcing application highly depends on whether a quality crowd can be recruited to undertake the corresponding tasks. Recent research has been focused on incentive mechanisms [2], [24] for mobile crowdsourcing, which determine how to recruit a crowd mainly based on their prices/costs. However, limited research efforts have been made to quality of the recruited crowd, i.e., quality of services/data each individual mobile user and the whole crowd are potentially capable of providing, which is the main focus of the paper. We aim to develop mathematical models to characterize the quality of a recruited crowd (a set of mobile users). We believe the models for Quality of Crowd (QoC) should be application-dependent and we introduce several such models to serve various applications. Furthermore, Unlike [2], [24], in our auction formulation, the bids are two-dimensional, which means the proof of mechanism properties in [2], [24] cannot be directly applied here; and we follow the Bayesian setting [17] (See Section III), which is a more realistic model.

We consider *fine-grained* MCS, in which each sensing task consists of multiple subtasks and a mobile user may make contributions to multiple subtasks. For example, if the goal of a sensing task is to cover a target area, then each subtask may corresponds to a sub-area. In this way, the recruited crowd may provide a better coverage for the target area. In addition, a sensing task may even include a set of heterogeneous subtasks. For example, subtask 1 may request the sensing crowd to collect WiFi signal strengths, while subtask 2 may request for signal strengths of cellular networks. Fine-grained MCS can lead to a better quality of service and allow a service user to specify a sensing task more flexibly. However, it also introduces additional complexity for crowd selection because a mobile user may be a good candidate for multiple subtasks, but may contribute differently to different subtasks. Existing incentive mechanisms [9] select the crowd for a single task, ignoring benefits that can be brought by sharing service/data with other tasks/subtasks. However, we aim to select a crowd to undertake a sensing task, while meeting a certain quality requirement (explained in Section II) for each of its subtasks. We summarize our contributions in the following:

- We introduce mathematical models for characterizing QoC for different sensing applications.
- Based on these models, we present a novel auction formulation for quality-aware and fine-grained MCS, which minimizes the expected expenditure subject to the quality requirement of each subtask.
- We discuss how to achieve the optimal expected expenditure, and present a practical incentive mechanism to solve the auction problem, which is shown to be truthful, individually rational and computationally efficient.
- We conducted trace-driven simulation using the mobility dataset of San Francisco taxies [18] and compared the proposed incentive mechanism with two well-designed baseline methods (rather than trivial random solutions). Extensive simulation results show the proposed mechanism achieves noticeable expenditure savings compared to the baselines; moreover, it produces close-to-optimal solutions.

II. Quality of Crowd (QoC) Models

TABLE I
MAJOR NOTATIONS

Notation	Explanation
i and M	Index of mobile users and the total number of mobile users
c_i and w_i	True and declared costs of mobile user i respectively
$\mathbf{Y}_i$ and $\mathbf{Z}_i$	True and declared quality score vectors of mobile user i respectively
$\mathbf{b}_i$ and $\mathbf{B}$	Bid of mobile user i and the corresponding vector
x_i and $\mathbf{x}$	Winner selection variable of mobile user i and the corresponding vector
p_i and $\mathbf{p}$	Payment to mobile user i and the corresponding vector
j and N	Index of subtasks and the total number of subtasks
r_j and $\mathbf{R}$	Quality requirement of subtask j and the corresponding vector
$g_j(\cdot)$	QoC model of subtask j

We focus on a general-purpose MCS system with a sensing crowd of M mobile users. A subset of mobile users will be recruited to undertake a sensing task including N subtasks. For each selected mobile user, there is a cost of c_i as explained above. A *quality score* is given for mobile user i participating in subtask j (denoted as y_{ij}), which quantifies the quality of services/data the mobile user is potentially capable of providing to that subtask. It is application-dependent and can be assigned according to various factors such as availability, accuracy of sensor data, reputation, etc. The cloud operator can calculate quality scores for mobile users and let them know their own quality scores. We use $\mathbf{Y}_i = [y_{i1}, ..., y_{ij}, ..., y_{iN}]$ to denote the quality score vector of mobile user i for all sensing subtasks.

The Quality of Crowd (QoC) for subtask j, q_j, quantifies the quality of services/data the sensing crowd is potentially capable of providing, which could be given by a function $g_j(\cdot)$ that can satisfy the following properties: 1) $g_j(\cdot)$ is a monotonically non-decreasing function of $\langle y_{ij} \rangle$ and $\langle x_i \rangle$, where x_i is a binary value indicating whether mobile user i is recruited or not; and 2) $g_j(\cdot)$ returns a value in $[0, 1]$. The first property reflects the nature that with a larger population of the recruited crowd and/or higher individual quality scores, the QoC for the corresponding subtask should not become worse. In order to make it easier for comparisons and understanding, the value of QoC should be scaled into $[0, 1]$, with 1 indicting the corresponding subtask can be perfectly completed by the recruited crowd. Note that the auction-based incentive mechanisms presented later are not restricted to any particular QoC model (function). In the following, we introduce several QoC models that can cover a large variety of sensing applications.

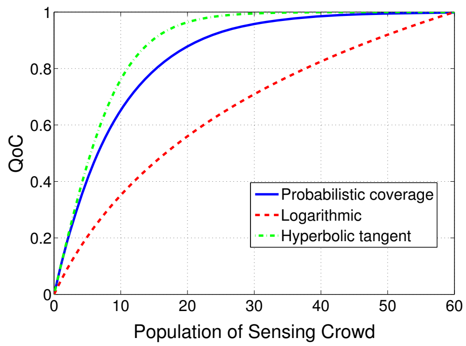


Fig. 2. QoC models

1) Linear model:

$$q_j = \frac{\min(\sum_i^M y_{ij}x_i, q_{\max})}{q_{\max}}. \quad (1)$$

This model simply sums up quality scores of all mobile users as the QoC if a goal $q_{\max}$ has not yet achieved; otherwise, the QoC remains at $q_{\max}$. This model is suitable for applications with a goal/constraint of achieving a certain sensing duration or collecting a certain number of samples. Here, y_{ij} can be the sensing duration or the number of sensing samples that mobile user i can potentially provide for subtask j. Linear models have been used in [7], [12].

2) Probabilistic coverage model:

$$q_j = 1 - \prod_i^M (1 - y_{ij}x_i). \quad (2)$$

If y_{ij} gives the probability that the target of subtask j (e.g., an area or a set of points of interest) can by covered by recruiting mobile user i, then q_j is the probability that the target can be covered by the recruited crowd. This model is suitable for sensing applications with a goal/constraint of covering a target area or a set of target points.

3) Logarithmic model:

$$q_j = \frac{log(1 + \sum_i^M log(1 + y_{ij}x_i))}{log(1 + \sum_i^M log(1 + y_{ij}))}. \quad (3)$$

In the numerator, the inner log term causes the return value to have a diminishing increment with the quality score, and the outer log term leads to diminishing increment with the population of the recruited crowd.

4) Hyperbolic tangent model:

$$q_j = tanh(\sum_i^M y_{ij}x_i). \quad (4)$$

Note that it has been shown by [24] that function (3) is submodular, i.e., the increase of the return value diminishes with the input set. We can easily show that function (4) is submodular too. These two models are suitable for most applications which extract meaningful information from sensor data, because usually given a larger data set, the additional information that can be obtained diminishes. Fig. 2 illustrates how QoC changes with the population of the crowd according to the three non-linear models. In this example, all mobile users have a common quality score of 0.1.

As mentioned above, we consider fine-grained MCS, in which each sensing task consists of multiple subtasks. Each subtask needs to be completed with a minimum quality requirement, r_j. We use $\mathbf{R} = [r_1, ..., r_j, ..., r_N]$ to denote a vector of quality requirements of all subtasks. The cloud operator recruits mobile users and makes sure $q_j = g_j(\mathbf{Y}, \mathbf{X}) \geq r_j, \forall j \in \{1, ..., N\}$, for the given sensing task, where $\mathbf{X} = [x_1, ..., x_i, ..., x_M]$.

III. Auction Formulation

In MCS, incentive mechanism design can be formulated as a *reverse* or *procurement* auction mechanism design problem. In the auction, 1) the cloud operator (the buyer) announces a sensing task to mobile users (bidders and sellers); 2) each mobile user i submits a bid $\mathbf{b}_i$ (defined below); 3) the cloud operator uses an incentive mechanism to select the winners and determine payments; 4) winners carry out the sensing task and submit results to the cloud operator; 5) the cloud operator checks the results and makes payments to winners. In the following, we use *mobile user* and *bidder* interchangeably.

Specifically, $\mathbf{b}_i = (w_i, \mathbf{Z}_i)$, in which w_i is mobile user i's declared cost, and $\mathbf{Z}_i$ is mobile user i's declared quality vector. If mobile user i does not want to participate in certain subtasks, the corresponding declared quality scores can be set to 0. Because mobile user i's true cost c_i and true quality vector $\mathbf{Y}_i$ are private and only known to mobile user i himself/herself, w_i and $\mathbf{Z}_i$ could be different from c_i and $\mathbf{Y}_i$, respectively. Different from [23], mobile users' private costs, $\langle c \rangle$, are assumed to follow a known distribution here. This assumption is known as *Bayesian setting* [17], and it is a realistic assumption because such a distribution can be obtained from historical data of previous auction transactions. $f_i(c)$ denotes the probability density function; and $F_i(c)$ denotes the corresponding cumulative distribution function. So $f_i(c) = \frac{d}{dc}F_i(c)$. $\mathbf{B} = [\mathbf{b}_1, ..., \mathbf{b}_i, ..., \mathbf{b}_M]$ is the bid vector of all mobile users. $\mathbf{B}_{-i}$ denotes the bids of all mobile users except i, so $\mathbf{B} = [\mathbf{b}_i, \mathbf{B}_{-i}]$. In addition, each mobile user i is a *single-minded bidder* [17], i.e., at a cost of c_i, he/she will participate in those subtasks, to which he/she has non-zero quality scores; or none at a cost of 0 otherwise.

The cloud operator must complete each subtask j to the required quality r_j, $\forall j \in \{1, ..., N\}$. Moreover, the cloud operator also wishes to conserve money and minimize its expected expenditure by selectively recruiting mobile users. Specifically, an auction mechanism takes the bid vector $\mathbf{B}$ and the quality requirement vector $\mathbf{R}$ as input and returns a winner vector $\mathbf{x} = [x_1, ..., x_i, ..., x_M]$, where $x_i = 1$ if mobile user i wins, and $x_i = 0$ otherwise; it also returns a payment vector $\mathbf{P} = [p_1, ..., p_i, ..., p_M]$, where p_i is the payment that the cloud operator will make to mobile user i. Based on the output of the auction, the *payoff* of mobile user i is defined as

$$u_i = \begin{cases} p_i - c_i, & x_i = 1; \\ 0, & x_i = 0. \end{cases} \tag{5}$$

The *expenditure* of a reverse auction is the sum of the payments $\sum_i^M p_i$ to all mobile users (bidders).

A. Desirable Properties

In this section, we describe three desirable properties for an auction mechanism:

- **Individual Rationality**: an auction mechanism is *individually rational* if for any bidder i, the payoff is non-negative when bidder i bids his/her true value $(c_i, \mathbf{Y}_i)$.
- **Truthfulness**: an auction mechanism is *truthful* if and only if for every bidder i and $\mathbf{B}_{-i}$, bidder i will not increase his/her payoff by making a bid $(w_i, \mathbf{Z}_i)$ that is different from his/her true value $(c_i, \mathbf{Y}_i)$; i.e., bidder i's payoff for bidding $(c_i, \mathbf{Y}_i)$ is at least his/her payoff for bidding any other bid $(w_i, \mathbf{Z}_i)$.
- **Computational Efficiency**: an auction mechanism is *computationally efficient* if the outcome can be computed in polynomial time.

Of the three properties, truthfulness is the most difficult to achieve. The bid is two-dimensional because for bidder i, the bid $\mathbf{b}_i$ contains two parts: bidder i's declared cost w_i and bidder i's declared quality vector $\mathbf{Z}_i$. As a result, Myerson's theorem [16] about the properties of one-parameter truthful mechanisms cannot be directly applied. To design a truthful auction mechanism with two dimensions, we introduce the following definitions:

Definition 1 (**w-Monotonicity**). *if bidder i wins by bidding $(w_i^*, (z_{i1}^*, ..., z_{ij}^*, ..., z_{iN}^*))$, then he/she also wins by bidding $(w_i', (z_{i1}^*, ..., z_{ij}^*, ..., z_{iN}^*))$ with any $w_i' \leq w_i^*$.*

Definition 2 (**z-Monotonicity**). *if bidder i wins by bidding $(w_i^*, (z_{i1}^*, ..., z_{ij}^*, ..., z_{iN}^*))$, then he/she also wins by bidding $(w_i^*, (z_{i1}', ..., z_{ij}', ..., z_{iN}'))$ with all $z_{ij}' \geq z_{ij}^*$.*

Definition 3 (**Critical Payment**). *the payment p_i for winning bidder i is set to the critical value d_i such that bidder i wins if $w_i < d_i$, and loses if $w_i > d_i$.*

Theorem 1. *An auction mechanism for MCS is truthful if it satisfies w-Monotonicity, z-Monotonicity and critical payment.*

The details of proof can be found in the appendix.

B. Virtual Cost

Next, we introduce *virtual cost* for reverse auctions and show its relationship with the expected expenditure. The concept of *virtual valuation* has been introduced for *forward auctions* in [16].

Definition 4 (Virtual Valuation)**.** *In a forward auction, the virtual valuation of bidder i with valuation v_i is*

$$\phi_i(v_i) = v_i - \frac{1 - F_i(v_i)}{f_i(v_i)}, \tag{6}$$

where the hazard rate $\frac{f_i(v_i)}{1-F_i(v_i)}$ *is assumed to be monotonically non-decreasing (*regularity assumption*).*

Theorem 2 ([17])**.** *Consider any (forward) truthful mechanism and fix the bids $\mathbf{b}_{-i}$ of all bidders except for bidder i. The expected payment of bidder i satisfies:*

$$E[p_i(v_i)] = E[\phi_i(v_i)x_i(v_i)]. \tag{7}$$

However, in a reverse auction, the valuation of a bidder can be treated as the negative of its cost, i.e., $v_i = -c_i$. Therefore,

$$\phi_i(v_i) = -c_i - \frac{1 - F_i(v_i)}{f_i(v_i)}$$

Moreover, it can be easily derived that

$$F_i(v_i) = 1 - F_i(c_i), \quad f_i(v_i) = f_i(-c_i) = f_i(c_i)$$

Hence, we have

$$\phi_i(v_i) = -(c_i + \frac{F_i(c_i)}{f_i(c_i)})$$

Definition 5 (Virtual Cost)**.** *In a reverse auction, the virtual cost of bidder i with cost c_i is*

$$\beta_i(c_i) = c_i + \frac{F_i(c_i)}{f_i(c_i)} \tag{8}$$

where the *regularity assumption* requires that $\frac{f_i(c_i)}{F_i(c_i)}$ is monotonically non-increasing. It is clear that $\beta_i(c_i) = -\phi_i(v_i)$.

Theorem 3. *Consider any reverse truthful mechanism and fix the bids $\mathbf{b}_{-i}$ of all bidders except for bidder i. The expected payment to bidder i satisfies:*

$$E[p_i(c_i)] = E[\beta_i(c_i)x_i(c_i)] \tag{9}$$

Proof: The payment from the buyer to a seller in a reverse auction can be viewed as the negative of the payment from a buyer to the seller in a forward auction. Therefore:

$$E[p_i(c_i)] = E[-p_i(v_i)] = E[-\phi_i(v_i)x_i(v_i)] = E[\beta_i(c_i)x_i(c_i)]$$

■

Because of Theorem 3 and the independence of all bidders' costs, it is fairly easy to show that the expected expenditure of a reverse truthful mechanism is equal to the total virtual cost. Therefore, to minimize the expected expenditure, it suffices to minimize the total virtual cost $\sum_i^M \beta_i(c_i)x_i$.

IV. Quality-aware Incentive Mechanisms (QIMs)

In this section, we present quality-aware incentive mechanisms (QIMs). First, we discuss how to achieve the optimal expected expenditure. Then, we present a practical QIM that is computationally efficient.

A. Optimal Solutions

The QIM design problem consists of two subproblems: *Winner Selection* and *Payment Determination*. Winner selection problem can be formulated as the following Integer Programming (IP) problem:
IP-Winner:

$$\min_{\mathbf{X}} \sum_{i=1}^{M} \beta_i(w_i) x_i \tag{10}$$

Subject to:

$$q_j = g_j(\mathbf{Z}, \mathbf{X}) \geq r_j, \quad \forall j \in \{1, ..., N\} \tag{11}$$

$$x_i \in \{0, 1\} \tag{12}$$

The objective (10) is to minimize total virtual cost, i.e., the expected expenditure of the cloud operator. Constraints (11) ensure that each subtask's quality requirement is met. Let $\Psi(\mathbf{B})$ denote the optimal value of IP-Winner and $\Psi(\mathbf{B}_{-i})$ denote the optimal value of IP-Winner with bid $\mathbf{b}_i$ removed. We can achieve the optimal as follows:

1) Winner Selection: Select winners $\mathbf{X}^*$ by solving IP-Winner;
2) Payment Determination: $p_i := \beta_i^{-1}(\Psi(\mathbf{B_{-i}}) - (\Psi(\mathbf{B}) - \beta_i(w_i)))$ if $x_i^* = 1$; $p_i := 0$, otherwise.

This incentive mechanism is designed by following the VCG (Vickrey-Clarke-Groves [17]) auction mechanism. It can be proven to be truthful and individually rational. The details of the proof can be found in the appendix. Note that both the Winner Selection and Payment Determination are different from those in [23].

However, solving IP-Winner may take exponentially long time for a large-sized problem instance. Even for the linear QoC model, IP-Winner is still an Integer Linear Problem (ILP), which is usually hard to solve. In our simulation, we used an optimization solver to provide optimal solutions for the linear QoC model. If we consider other QoC models, then IP-Winner becomes a non-linear integer programming problem, which is even much harder. In addition to time complexity, it has been shown that truthfulness cannot be preserved by a VCG-based auction mechanism with an approximation (instead of optimal) algorithm [17]. Therefore, we present a non-VCG-based QIM with computational efficiency.

B. Computationally Efficient QIM

Here, we present a QIM that is truthful, individually rational and computationally efficient, which we call *QIM-E*. Similar to the above method, QIM-E consists of two phases: *Winner Selection* and *Payment Determination*. *Winner Selection* (Algorithm 1) is a heuristic approach, which keeps selecting the mobile user (bidder) with the smallest weight as a winner. We adopt the following metric α_i as the weight to assist the selection:

$$\alpha_i = \frac{\beta(w_i)}{\sum_{j=1}^{N} \frac{v_{ij}}{r_j}}, \tag{13}$$

where v_{ij} is the *marginal* quality score mobile user i can contribute to subtask j (according to a QoC model), given a prior winner set (i.e., crowd) S':

$$v_{ij} = \begin{cases} \min(g_j(S' \cup \{i\}), r_j) - g_j(S'), & g_j(S') < r_j; \\ 0, & \text{otherwise.} \end{cases}$$

Then the algorithm updates $\langle v_{ij} \rangle$ and $\langle \alpha_i \rangle$ because in each iteration, $g_j(S')$ changes with the newly selected mobile user. For those remaining mobile users with a marginal quality score of 0 for all subtasks, they will be eliminated from the auction. The algorithm stops when there are no mobile users left.

Algorithm 1: Winner Selection of QIM-E

Input : Bid vector $\mathbf{B}$, mobile user (bidder) set $\mathbf{S}$, subtask QoC models $\langle g_j(\cdot) \rangle$ and quality requirement vector $\mathbf{R}$
Output: Winner vector $\mathbf{X}$

$x_i := 0$, $\forall i \in \{1, \cdots, M\}$;
$\mathbf{S}' := \emptyset$;
while $\mathbf{S} \neq \emptyset$ **do**
  $\alpha_i := \frac{\beta(w_i)}{\sum_{j=1}^{N} \frac{v_{ij}}{r_j}}$, $\forall i \in \mathbf{S}$;
  $k := \arg\min_{i \in \mathbf{S}}(\alpha_i)$;
  $x_k := 1$;
  $\mathbf{S} := \mathbf{S} \setminus \{k\}$;
  $\mathbf{S}' := \mathbf{S}' \cup \{k\}$;
  update $\langle v_{ij} \rangle$, $\forall i \in \mathbf{S}$;
  forall the $m \in \mathbf{S}$ **do**
    if $v_{mj} = 0$, $\forall j \in \{1, ..., N\}$ **then**
      $\mathbf{S} := \mathbf{S} \setminus \{m\}$;
return $\mathbf{X} := [x_1, ..., x_i.., x_M]$;

Note that the weight of each remaining mobile user changes in each iteration; so instead of maintaining a fixed sorted bidder list, the algorithm updates their weights and makes the selection based on the updated weights. In *Payment Determination* (Algorithm 2), to determine the payment for winning mobile user l, the algorithm repeats the above winner selection for the mobile user set with l excluded. When a winning mobile user k is found such that his/her selection can disqualify l from winning the auction, the payment is set to the highest cost that helps l disqualify k or any winning mobile user before k.

C. Proof of Properties

Next, we show that QIM-E is truthful, individually rational, and computationally efficient.

Lemma 1. *w-Monotonicity and z-Monotonicity are preserved in the Winner Selection of QIM-E.*

Proof: Suppose mobile user i wins by bidding $(w_i^*, \mathbf{Z}_i^*) = (w_i^*, (z_{i1}^*, ..., z_{ij}^*, ..., z_{iN}^*))$. We prove that: 1) he/she will also win by bidding $(w_i', \mathbf{Z}_i^*) = (w_i', (z_{i1}^*, ..., z_{ij}^*, ..., z_{iN}^*))$ with any $w_i' < w_i^*$; and 2) he/she will also win by bidding $(w_i^*, \mathbf{Z}_i') = (w_i^*, (z_{i1}', ..., z_{ij}', ..., z_{iN}'))$ with all $z_{ij}' > z_{ij}^*$.

Algorithm 2: Price Determination of QIM-E

Input : Bid vector $\mathbf{B}$, mobile user (bidder) set $\mathbf{S}$, subtask QoC models $\langle g_j(\cdot)\rangle$, quality requirement vector $\mathbf{R}$, winner vector $\mathbf{X}$
Output: Payment vector $\mathbf{P}$
1 **forall the** $l \in \mathbf{S}$ **do**
2   $p_l := 0$;
3   **if** $x_l = 1$ **then**
4     $\mathbf{S}^* := \mathbf{S} \setminus \{l\}$;
5     $\mathbf{S}' := \emptyset$;
6   calculate $\langle v_{lj}\rangle$;
7   **while** $\mathbf{S}^* \neq \emptyset$ **do**
8       $\alpha_i := \frac{\beta(w_i)}{\sum_{j=1}^{N} \frac{v_{ij}}{r_j}}, \forall i \in \mathbf{S}^*$;
9       $k := \arg\min_{i \in \mathbf{S}^*}(\alpha_i)$;
10       $\mathbf{S}^* := \mathbf{S}^* \setminus \{k\}$;
11       $\mathbf{S}' := \mathbf{S}' \cup \{k\}$;
12       $p_l := \max(p_l, \beta_l^{-1}(\alpha_k \sum_{j=1}^{N} \frac{v_{lj}}{r_j}))$;
13   update $\langle v_{ij}\rangle, \forall i \in \{\mathbf{S}^*, l\}$;
14   **if** $v_{lj} = 0, \forall j \in \{1, ..., N\}$ **then**
15   break;
16   **forall the** $m \in \mathbf{S}^*$ **do**
17   **if** $v_{mj} = 0, , \forall j \in \{1, ..., N\}$ **then**
18           $\mathbf{S}^* := \mathbf{S}^* \setminus \{m\}$;
19 **return** $\mathbf{P} := [p_1, ..., p_i, ..., p_M]$;

Let α_i^*, $\langle v_{ij}^*\rangle$ denote the weight and marginal quality scores respectively when mobile user i bids $(w_i^*, \mathbf{Z}_i^*)$. Let α_i' and $\langle v_{ij}'\rangle$ denote the weight and marginal quality scores respectively when he/she bids $(w_i', \mathbf{Z}_i^*)$ or $(w_i^*, \mathbf{Z}_i')$. In either case of $(w_i', \mathbf{Z}_i^*)$ or $(w_i^*, \mathbf{Z}_i')$, it is clear that $v_{ij}' \geq v_{ij}^*$ and $\alpha_i' < \alpha_i^*$; i.e., the weight becomes smaller in each iteration for him/her. Moreover, as illustrated in lines 10–12 in Algorithm 1, if he/she has not been eliminated by bidding $(w_i^*, \mathbf{Z}_i^*)$, he/she will not be eliminated by bidding $(w_i', \mathbf{Z}_i^*)$ or $(w_i^*, \mathbf{Z}_i')$ either. Therefore, he/she will still win with bid $(w_i', \mathbf{Z}_i^*)$ or $(w_i^*, \mathbf{Z}_i')$. This completes the proof. ■

Lemma 2. *The payment p_l is set to a critical value for each winning mobile user (bidder) l in QIM-E.*

Proof: Let k be the index of mobile user with the smallest weight in each iteration until his/her selection disqualifies mobile user l. Let $d_l = \max_k(\beta_l^{-1}(\alpha_k \sum_{j=1}^{N} \frac{v_{lj}}{r_j}))$. Note that the marginal quality scores $\langle v_{lj}\rangle$ are updated in each iteration. If mobile user l bids $w_l > d_l$, then $\alpha_l > \alpha_k, \forall k$, meaning l does not have the smallest weight in any iteration before he/she is disqualified by k and thus will be eliminated from the auction. If mobile user l bids $w_l < d_l$, then $\alpha_l < \alpha_k$ in one or more iterations, meaning l will be chosen as a winner the first time when $\alpha_l < \alpha_k$ happens. Hence d_l is the critical value for winning mobile user l. In Algorithm 2, the payment p_l is set to d_l. This completes the proof. ■

Theorem 4. *QIM-E is truthful.*

Proof: According to Lemmas 1 and 2 as well as Theorem 1, QIM-E is truthful. ■

Theorem 5. *QIM-E is individually rational.*

Proof: We examine two possible cases. First, it is clear that the payoff of mobile user l is 0 if mobile user l is not a winner according to Algorithm 2. Second, if mobile user l is a winner, let the critical value be d_l and mobile user l's cost be c_l. Since QIM-E preserves the critical payment property as shown in Lemma 2, it is obvious that $w_l < d_l$ and $d_l = p_l$. Since $w_l = c_l$ in a truthful mechanism, it is clear that $p_l - c_l > 0$. Therefore, the payoff is always non-negative. This completes the proof. ■

Theorem 6. *QIM-E is computationally efficient.*

Proof: In Algorithm 1, line 4 takes $O(MN)$ time to calculate $\langle \alpha_i\rangle$ and update $\langle v_{ij}\rangle$. Note that finding the mobile user with the minimum weight only takes $O(M)$ in line 5. Since the while-loop runs M times, the time complexity of Algorithm 1 is $O(M^2N)$.

However, in Algorithm 2, the for-loop (lines 1–18) iterates M times, and the inner while-loop (lines 7–18) takes $O(M^2N)$ time because it has the same complexity with Algorithm 1. So Algorithm 2 takes $O(M^3N)$ time. Therefore, the overall time complexity of QIM-E is $O(M^3N)$, which completes the proof. ■

V. Performance Evaluation

In this section, we present and discuss simulation results based on real data to justify the effectiveness of the proposed mechanisms.

A. Baseline Methods

For fair comparisons, we chose two well-designed incentive mechanisms (one of them is truthful and individually rational) as the baselines, instead of trivial random solutions. The first baseline is a revised version of the greedy method with a fixed list of bidders (referred to as Fix-L) presented in [25]. Since we deal with a two-parameter auction (cost and quality score), we use $\alpha_i = \frac{\beta(w_i)}{\sum_{j=1}^{N} z_{ij}}$ as the weight to sort the bidders in non-decreasing order to obtain the fixed list. Then we iterate through the fixed list and select winners until the quality requirements of all subtasks are met. The winners are paid based on the corresponding critical values [25]. Similar as in [25], it can be shown that Fix-L preserves truthfulness and individual rationality.

In the second baseline approach, all bidders are sorted in the non-decreasing order based on their virtual cost (referred to as Low-C). The algorithm repeatedly selects the bidder with the lowest virtual cost among the remaining bidder set. This process stops when quality requirements of all subtasks are met and winners are paid with critical values. Note that even though this approach is not truthful, it is still a good baseline to compare with because the cloud operator tends to directly reduce the expenditure by selecting bidders with low costs.

B. Simulation Settings

We conducted trace-driven simulation for performance evaluation using the mobility dataset [18] of San Francisco taxies, which contains GPS coordinates of approximately 500 taxis collected over 30 days in the San Francisco Bay Area. For the distributions of mobile user (bidder) costs, we considered the uniform distribution $f_i(c_i) = 0.25$ in the range of $(0, 4]$, the exponential distribution $f_i(c_i) = 0.5e^{-0.5c_i}$ in the range of $(0, +\infty)$ and χ^2-distribution with freedom degree of 2. Note that these functions have the same mean value of 2 and the first two distributions were also used in [11] and [25]. For QoC models of subtasks, we have implemented the linear model, the probabilistic coverage model and the hyperbolic tangent model introduced in Section II. In our simulation, each subtask corresponds a sub-area, each of which is a square-like region with a randomly chosen center, whose left/top and right/bottom boundaries differ by 0.0005 degrees in both longitude and latitude (about 160 feet). We derived the quality score of each taxi i for a subtask j by dividing the number of samples of i within sub-area j by the number of weeks i showed up in the dataset, which captures the availability of the mobile user. Due to non-uniform distribution of samples, to ensure the quality requirements are satisfied, we normalized them by a large number, 50, and curved them with an upper and lower bounds of 0.15 and 0.04 respectively.

We came up with the following scenarios for simulation. Simulation runs were conducted on a computer with a 2.2GHz Intel Core i7 CPU and 16GB memory. When the linear QoC model was used, the optimal expected expenditures were obtained using the method presented in Section IV-A (labeled as *QIM-Opt*), in which Gurobi Optimizer [6] was employed to solve the corresponding ILP problems. Each number presented here is an average over 30 runs.

1) In scenarios 1 and 2, the number of subtasks was fixed to 15; quality requirements of subtasks were set to be uniformly distributed in $[0.7, 0.8]$. In scenario 1, the linear model was applied for QoC; the number of mobile users was varied for all the cost distributions described above. In scenario 2, the above exponential distribution was applied for costs; the number of mobile users was varied for the three QoC models mentioned above. The results of scenario 1 are presented in Fig. 3 and results of scenario 2 are shown in Fig. 3(a) and Fig. 4.

2) In scenario 3, the linear model was applied for QoC; costs of mobile users were generated by following the above exponential distribution; the number of mobile users was set to 350. The number of subtasks was increased from 5 to 30 with a step size of 5. The corresponding results are presented in Fig. 5.

3) In scenario 4, we evaluated the running time of proposed mechanisms. The number of subtasks was fixed to 15; the linear model was applied for QoC; costs of mobile users were generated according to the above exponential distribution; quality requirements of subtasks were uniformly distributed in $[0.7, 0.8]$. The number of mobile users was increased from 250 to 500 with a step size of 50. The corresponding results are presented in Fig. 6.

C. Simulation Results and Analysis

We can make the following observations from the results.

1) In Fig. 3, we show the expected expenditures under different cost distributions, when the linear model was applied for QoC. In Fig. 5, we show how the expected expenditure changes with the number of subtasks. From these figures, we can see that the expected expenditures given by QIM-E are consistently close to the optimal values. Specifically, in Fig. 3, QIM-E produces only 3.9%, 5.1% and 4.4% more expenditures than the optimal for the exponential, uniform and χ^2-distributions of costs on average respectively. Moreover, in Fig. 5, QIM-E gives only 3.2% more expenditures than the optimal on average.

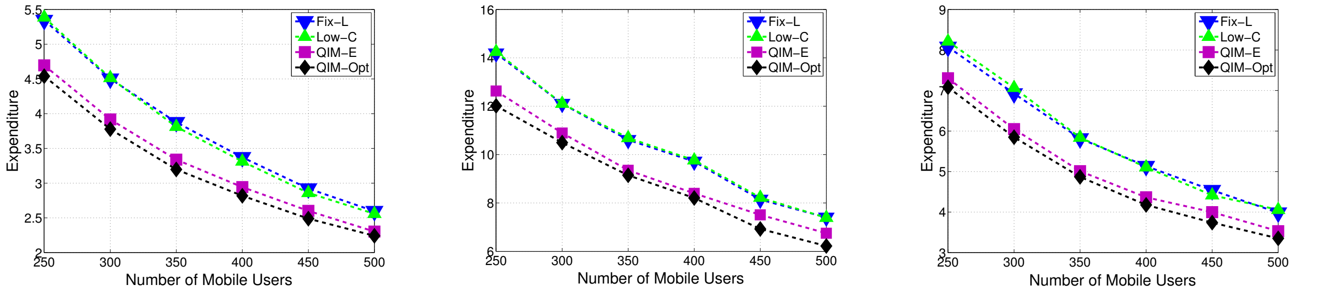
2) From Figs. 3–5, we can see that QIM-E consistently outperforms Fix-L and Low-C. The reason is that when selecting winners, Low-C does not carefully consider the quality scores of the mobile users. Even though Fix-L considers the individual quality scores, it doesn't carefully take QoC into consideration . On the contrary, QIM-E favors those mobile users who contribute the most marginal QoC. Specifically, in Fig. 3, QIM-E produces about 11.9%, 10.7%, 12.6% less expenditures than Fix-L for the exponential, uniform and χ^2-distributions of costs on average respectively. Moreover, in Figs. 3(a) and 4, QIM-E produces about 11.9%, 13.2%, 10.6% less expenditures than Fix-L for the linear, probabilistic coverage and hyperbolic tangent model of QoC respectively. Similar observations can be made from Fig. 5. Note that the performance of Low-C is very close to Fix-L in all the scenarios.

3) Monotonicity can be observed in Figs. 3–5. As expected, in Figs. 3 and 4, with more mobile users to choose from, all mechanisms yield lower expenditures. On the contrary, we can see that more subtasks lead to higher expenditures no matter which mechanism is used according to Fig. 5.

4) Fig. 6 shows the running time of different mechanisms with various numbers of mobile users. The running time of QIM-E is only 8.8% of that of QIM-Opt on average, which shows QIM-E is scalable. The running times of QIM-E, Fix-L and Low-C are fairly close to each other, which matches the theoretical analyses that suggest they all have a time complexity of $O(M^3N)$.

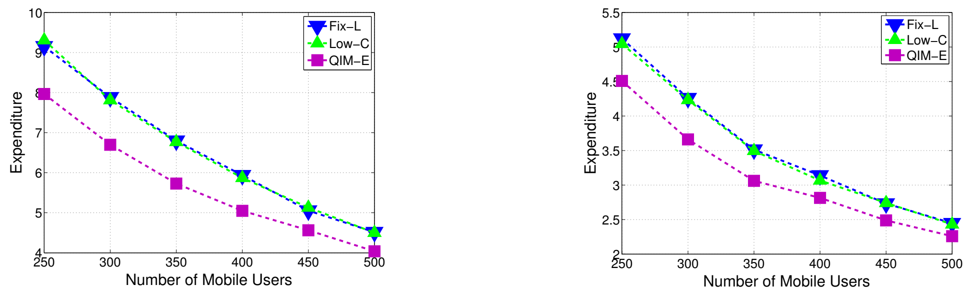
VI. Related Work

Research efforts have been made to develop general-purpose MCS systems, such as PRISM [1] and Medusa [19]. Incentive mechanism design has been addressed in the context general MCS systems recently. Yang *et al.* introduced two models for MCS: platform-centric and user-centric; and designed an incentive mechanism using a Stackelberg game for the platform-centric model as well as an auction-based incentive mechanism for the user-centric model in [24]. Duan *et al.* proposed a reward-based collaboration mechanism in [2], in which collaborators share a total reward announced by the client. In addition, they investigated how the client can



(a) Exponential distribution (b) Uniform distribution (c) χ^2-distribution

Fig. 3. Performance with the linear QoC model and different cost distributions (Scenario 1)



(a) Probability coverage model (b) Hyperbolic tangent model

Fig. 4. Performance with different QoC models and the exponential cost distributions (Scenario 2)

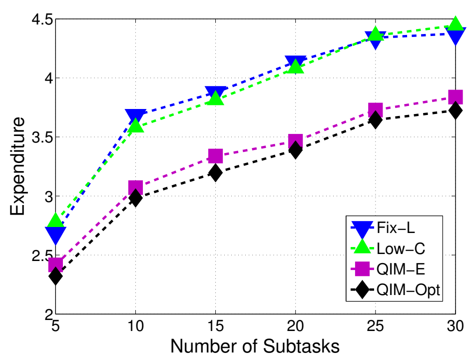


Fig. 5. Performance with different numbers of subtasks (Scenario 3)

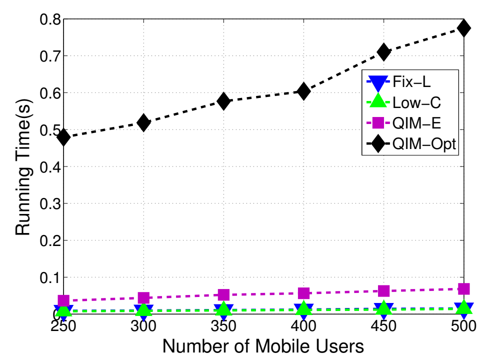


Fig. 6. Running time (Scenario 4)

design an optimal contract by specifying different task-reward combinations for different user types. In [26], Zhao *et al.* considered the scenario where mobile users arrive one by one online in a random order. They presented two online incentive mechanisms, in which mobile users submit their private types to the crowdsourcer when arrive and the crowdsourcer aims to select a user subset for maximizing a utility function with a budget constraint. Feng *et al.* presented a reverse auction framework named TRAC in [4] to model location based auction interactions between a cloud and smartphones, which minimizes the social cost. In [5], the authors presented two truthful incentive mechanisms for both the offline and online cases, given dynamic smartphones, uncertain arrivals of tasks, strategic behaviors and private information of smartphones. In [27], the authors first designed an incentive mechanism, EFF, which eliminates dishonest behavior with the help from a trusted third party for arbitration. They then designed another mechanism DFF, which, without the help from any third party, discourages free-riding and false-reporting.

Recently, several research works have addressed incentive mechanism design with quality considerations. In [9], Koutsopoulos *et al.* seeked a mechanism for user participation level determination and payment allocation which minimizes the total cost of compensating participants, while delivering a certain quality of experience to service requesters. They designed a mechanism that optimally solves this problem. In [7], the authors presented an approximation mechanism to find an efficient task allocation with quality of sensing requirements as well as a pricing mechanism based on bargaining theory. Luo *et al.* designed an incentive mechanism [14] based on all-pay auctions to attract contributions from mobile users. In [12], Jin *et al.* designed a truthful, individually rational and computationally efficient mechanism that approximately maximizes the social welfare for single-minded combinatorial models, which was shown to have an approximation ratio, assuming a linear quality model. Moreover, they designed an iterative descending mechanism with individual rationality for multi-minded combinatorial models.

We summarize the differences between our work and these related works in the following: 1) Unlike most related works, we consider fine-grained MCS, in which a sensing task consists of multiple subtasks and a mobile user may make contributions to multiple subtasks. 2) Many related works, such as [2], [4], [5], [24], [26], [27], have not offered careful consideration for QoC and quality requirements of subtasks, which, however, is the main focus of this paper. 4) The auction formulation here (with the objective of minimizing the expected expenditure subject to quality requirements) is mathematically different from those in related works [7], [12], [9], [14]. 5) Unlike some previous works mainly focusing on a specific quality model [7], [12] (such as the linear model), we conduct a comprehensive study for QoC models.

VII. Conclusions

In this paper, we have studied incentive mechanism design for quality-aware and fine-grained MCS. First, we have introduced several models to characterize QoC for different sensing applications. Based on these models, we have presented a novel auction formulation for quality-aware and fine-grained MCS, which minimizes the expected expenditure subject to the quality requirement of each subtask. We have discussed how to achieve the optimal expected expenditure, and presented a practical incentive mechanism to solve the auction problem, which has been shown to be truthful, individual rational and computational efficient. We have conducted trace-driven simulation using the mobility dataset of San Francisco taxies. Extensive simulation results have shown the proposed incentive mechanism achieves noticeable expenditure savings compared to two well-designed baseline methods, and moreover, it produces close-to-optimal solutions.

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VIII. Appendix

As discussed in Section III-A, we show that an MCS auction mechanism is truthful if it has the w-Monotonicity, z-Monotonicity and critical payment properties.

Lemma 3. *In an MCS auction mechanism, if w-Monotonicity, z-Monotonicity, and critical payment are satisfied, bidder i will not increase his/her payoff by bidding $(c_i, \mathbf{Z}_i) = (c_i, (z_{i1}, ..., z_{ij}, ..., z_{iN}))$ instead of $(c_i, \mathbf{Y}_i) = (c_i, (y_{i1}, ..., y_{ij}, ..., y_{iN}))$, when $\mathbf{Y}_j \neq \mathbf{Z}_j$.*

Proof: We examine two possible scenarios:

1) $z_{ij} < y_{ij}$ for every j. Let d_y, d_z denote the critical payments for bidding $(c_i, \mathbf{Y}_i)$ and $(c_i, \mathbf{Z}_i)$ respectively. We consider two sub-cases: $a)$ bidder i wins by bidding $(c_i, \mathbf{Z}_i)$. Based on z-Monotonicity, we know that he/she will also win by bidding $(c_i, \mathbf{Y}_i)$. In other words, for any $c_i < d_z$, we have $c_i < d_y$. Hence, $d_y \geq d_z$; the payment of bidding $(c_i, \mathbf{Y}_i)$ will not be decreased. $b)$ bidder i loses by bidding $(c_i, \mathbf{Z}_i)$. In this sub-case, the payoff of bidding $(c_i, \mathbf{Y}_i)$ is 0 if he/she loses and non-negative if he/she wins.

2) $z_{ij} > y_{ij}$ for one or more j's. Before actually making payments to bidder i, the cloud operator has a quality control that makes sure the actual quality score y_{ij} (derived from the submitted sensor data) is equal to or greater than z_{ij}. If not, no payments will be made to bidder i, yielding negative payoff for him/her with bidding $(c_i, \mathbf{Z}_i)$.

The above two cases complete the proof. ∎

Lemma 4. *In an MCS auction mechanism, if w-Monotonicity, z-Monotonicity, and critical payment are satisfied, bidder i will not increase his/her payoff by bidding $(w_i, \mathbf{Z}_i)$ instead of $(c_i, \mathbf{Z}_i)$, when $c_i \neq w_i$.*

Proof: Denote the Critical Payment for bidding $(c_i, \mathbf{Z}_i)$ by d. We consider two cases:

1) $(c_i, \mathbf{Z}_i)$ is a losing bid. In this case, $c_i > d$. We consider two sub-cases: $a)$ $(w_i, \mathbf{Z}_i)$ is a losing bid. Bidder i would have a 0 payoff, which is not better than bidding $(c_i, \mathbf{Z}_i)$. $b)$ $(w_i, \mathbf{Z}_i)$ is a winning bid. He/she receives the payment d because the critical payment is independent of w_i; the payoff of bidding $(w_i, \mathbf{Z}_i)$ would be negative, since $d < c_i$.

2) $(c_i, \mathbf{Z}_i)$ is a winning bid. If $(w_i, \mathbf{Z}_i)$ is a winning bid, bidder i receives the same payment p with $(c_i, \mathbf{Z}_i)$. If $(w_i, \mathbf{Z_i})$ is a losing bid, he/she receives a payment of 0.

The above two cases complete the proof. ∎

Theorem 1. *An auction mechanism for MCS is truthful if it satisfies w-Monotonicity, z-Monotonicity and critical payment.*

Proof: Based on the definition of truthfulness, it suffices to show that bidder i will not increase his/her payoff by bidding any other bid $(w_i, \mathbf{Z}_i)$ instead of $(c_i, \mathbf{Y}_i)$. Lemma 4 has shown that bidder i will not increase his/her payoff by bidding $(w_i, \mathbf{Z}_i)$ instead of $(c_i, \mathbf{Z}_i)$. In Lemma 3, we have proved that bidder i will not increase his/her payoff by bidding $(c_i, \mathbf{Z}_i)$ instead of $(c_i, \mathbf{Y}_i)$. Therefore, bidder i will not increase his/her payoff by bidding any $(w_i, \mathbf{Z}_i)$ instead of $(c_i, \mathbf{Y}_i)$. This completes the proof. ∎

Next, we show that QIM with the optimal expected expenditure discussed in Section IV-A(referred to as QIM-Opt) is truthful and individually rational. To prove the truthfulness, we show w-Monotonicity, z-Monotonicity and critical payment properties are preserved in QIM-Opt.

Lemma 5. *w-Monotonicity is satisfied by the Winner Selection of QIM-Opt.*

Proof: Suppose mobile user i wins by bidding $\mathbf{b}_i^* = (w_i^*, (z_{i1}^*, ..., z_{ij}^*, ..., z_{iN}^*))$, or equivalently $\Psi(\mathbf{B}_{-i}) > \Psi((\mathbf{b}_i^*, \mathbf{B}_{-i}))$. We will prove that mobile user i also wins by bidding $\mathbf{b}_i' = (w_i', (z_{i1}^*, ..., z_{ij}^*, ..., z_{iN}^*))$ with any $w_i' < w_i^*$ through contradiction. Suppose mobile user i will lose by bidding $\mathbf{b}_i'$. Then $\Psi((\mathbf{b}_i', \mathbf{B}_{-i})) = \Psi(\mathbf{B}_{-i})$. Therefore, $\Psi((\mathbf{b}_i', \mathbf{B}_{-i})) > \Psi((\mathbf{b}_i^*, \mathbf{B}_{-i}))$. However, with the same winner vector $\mathbf{x}$ of $(\mathbf{b}_i^*, \mathbf{B}_{-i})$, the total virtual cost of $(\mathbf{b}_i^*, \mathbf{B}_{-i})$ would be greater than $(\mathbf{b}_i', \mathbf{B}_{-i})$ because $w_i^* > w_i'$; this contradicts the statement that $\Psi((\mathbf{b}_i', \mathbf{B}_{-i})) > \Psi((\mathbf{b}_i^*, \mathbf{B}_{-i}))$. Hence, the supposition is false and mobile user i will also win by bidding $\mathbf{b}_i'$. This completes the proof. ∎

Lemma 6. *z-Monotonicity is satisfied by the Winner Selection of QIM-Opt.*

Proof: Suppose that mobile user i wins by bidding $\mathbf{b}_i^* = (w_i^*, (z_{i1}^*, ..., z_{ij}^*, ..., z_{iN}^*))$, or equivalently $\Psi(\mathbf{B}_{-i}) > \Psi((\mathbf{b}_i^*, \mathbf{B}_{-i}))$. We will prove that mobile user i also wins by bidding $\mathbf{b}_i' = (w_i^*, (z_{i1}', ..., z_{ij}', ..., z_{iN}'))$ with all $z_{ij}' \geq z_{ij}^*$ through contradiction. Suppose mobile user i will lose by bidding $\mathbf{b}_i'$. Then $\Psi((\mathbf{b}_i', \mathbf{B}_{-i})) = \Psi(\mathbf{B}_{-i})$. Therefore, $\Psi((\mathbf{b}_i', \mathbf{B}_{-i})) > \Psi((\mathbf{b}_i^*, \mathbf{B}_{-i}))$. However, with the same winner vector $\mathbf{x}$ of $(\mathbf{b}_i^*, \mathbf{B}_{-i})$, the total virtual cost of $(\mathbf{b}_i', \mathbf{B}_{-i})$ is equal to the total virtual cost of $(\mathbf{b}_i^*, \mathbf{B}_{-i})$; this contradicts the statement that $\Psi((\mathbf{b}_j', \mathbf{B}_{-i})) > \Psi((\mathbf{b}_i^*, \mathbf{B}_{-i}))$. Hence, the supposition is false and mobile user i will also win by bidding $\mathbf{b}_i'$. This completes the proof. ∎

Lemma 7. *$p_i = \beta_i^{-1}(\Psi(\mathbf{B}_{-i}) - (\Psi(\mathbf{B}) - \beta_i(w_i)))$ is a critical value for winning mobile user i in QIM-Opt.*

Proof: In QIM-Opt for winning mobile user i, $\Psi(\mathbf{B}_{-i}) - (\Psi(\mathbf{B}) - \beta_i(w_i))$ is calculated based on the opportunity cost, which is the increment of total virtual cost of other mobile users caused by the absence of mobile user i. The opportunity cost in a reverse auction corresponds to the concept of opportunity cost in a forward auction introduced in [17]. Because of the regularity assumption, $\beta_i(\cdot)$ is a monotonically increasing function. Therefore if $w_i > \beta_i^{-1}(\Psi(\mathbf{B}_{-i}) - (\Psi(\mathbf{B}) - \beta_i(w_i)))$, it will result in a virtual cost higher than the opportunity cost. So mobile user i will not be selected as a winner. Otherwise if $w_i < \beta_i^{-1}(\Psi(\mathbf{B}_{-i}) - (\Psi(\mathbf{B}) - \beta_i(w_i)))$, it will yield a virtual cost lower than the opportunity cost. So mobile user i will be selected as a winner. This completes the proof. ∎

Theorem 7. *QIM-Opt is truthful.*

Proof: According to Lemmas 5, 6, and 7 as well as Theorem 1, QIM-Opt is truthful. This completes the proof. ∎

Theorem 8. *QIM-Opt is individually rational.*

Proof: If mobile user i bids true value $(c_i, \mathbf{Y}_i)$, his/her payoff is $u_i = p_i - c_i = \beta_i^{-1}(\Psi(\mathbf{B_{-i}}) - \Psi(\mathbf{B}) + \beta_i(c_i)) - c_i$. The optimality of $\Psi(\mathbf{B})$ causes $\Psi(\mathbf{B_{-i}}) - \Psi(\mathbf{B}) \geq 0$. Moreover, since $\beta_i(\cdot)$ is monotonically increasing, we have $u_i \geq 0$. This completes the proof. ∎